Remote Estimation under Energy-Aware Multisensor Admission Control over Unreliable Channels

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***Abstract*— We study remote state estimation in wireless networked control systems with energy-constrained sensors sharing an unreliable channel. Unlike traditional approaches assuming fixed sensor participation, we introduce a population-based framework for *dynamic sensor admission control*, modeling active, idle, and depleted sensors and accounting for collision-induced packet losses. We formulate a finite-horizon optimal control problem, solved analytically for a shared channel with constant reliability via a linear-quadratic admission policy, highlighting trade-offs between communication effort and estimation accuracy. When multiple sensors transmit simultaneously, collisions increase packet error probability. Motivated by this, we numerically derive an optimal admission policy to show how adaptive admission control improves estimation under energy constraints. Simulations demonstrate that the framework balances estimation accuracy, communication effort, and energy consumption, prolonging sensor lifetime.**

***Index Terms*— remote estimation, unreliable channels, multisensor admission control.**

I. Introduction

The rapid penetration of sensors across diverse applications has created both new opportunities and significant challenges for remote state estimation. Such applications include target tracking and mapping in autonomous and cooperative robotics [1], traffic monitoring in intelligent transportation systems [2], and many others. Compared with traditional hardwire systems, wireless networked control systems (WNCSs) provide notable advantages, including cost efficiency, flexibility, and ease of deployment [3]. In such systems, a remote estimator relies on real-time sensor data to obtain an accurate estimate of the system being monitored. However, wireless communication links are often unreliable due to fading, interference, and congestion leading to packet losses and delays, which can degrade estimation accuracy and system stability if not properly handled [4]–[6].

For single sensor setups, existing works have explored sensor scheduling techniques that jointly consider communication constraints and estimation objectives [7]–[10]. In particular, [7] focused on scheduling for a single sensor transmitting reliably to a remote estimator, aiming to minimize the terminal error covariance over a finite horizon. Later, [8] proved the optimality of this policy. The works of [9], [10] extended this framework to two sensors and considered energy constraints and temporally correlated channel variations, triggering further extensions to multi-sensor scenarios.

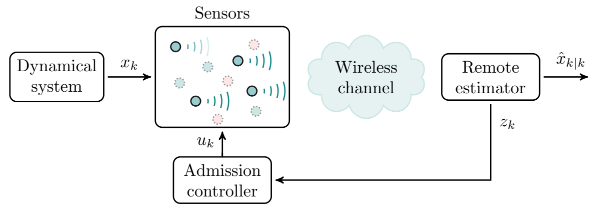

Fig. 1. Remote estimation and admission control scheme.

Towards this extension, several works have studied multisensor scheduling for remote estimation under communication and energy constraints. In [11], scheduling over a packet-dropping channel with energy-limited sensors competing for shared access was considered, with the optimal policy exhibiting a threshold structure based on estimation error covariance. The work in [12] formulated multi-process scheduling as an infinite-horizon Markov decision process (MDP) and proved the existence of an optimal deterministic stationary policy. The work in [13] was among the first to consider remote estimation using multiple sensors over a collision channel, where optimal remote estimation is achieved through deterministic threshold transmission policies at the sensors under a mean-squared error criterion. More recently, [14] showed that receivers capable of multi-packet reception can benefit from simultaneous transmissions, improving estimation accuracy and energy efficiency. Extending these results, [15] proposed a stochastic event-triggered scheduler, showing that the system state follows a Gaussian mixture and deriving efficient approximate minimum mean squared error (MMSE) estimators to handle the exponential complexity.

While these studies provide valuable insights into sensor scheduling, they generally assume fixed sensor participation or do not explicitly account for sensor availability dynamics under collision-prone channels. In this work, we address these gaps by introducing a *dynamic sensor admission control* framework for remote state estimation over shared, unreliable wireless channels with energy-constrained sensors (see Fig. 1 for a system overview). We model the evolution of active, idle, and depleted sensors within a population-based framework, and integrate a lossy channel model that accounts for interference by increasing packet loss as more sensors transmit their measurements concurrently. This formulation allows us to cast the problem as a finite-horizon optimal

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control problem and solve it to balance estimation accuracy, network congestion, and energy depletion. Building on this formulation, the main contributions are as follows:

1) We formulate the admission of energy-constrained sensors over a shared unreliable channel as a finite-horizon optimization problem, determining the number of active transmitting sensors at each iteration.
2) For unreliable channels, where packet success depends on simultaneous transmissions, we analyze the resulting nonlinear structure and show how the optimal policy can be efficiently computed numerically, highlighting the impact of collisions on sensor admission. Moreover, we derive a closed-form linear–quadratic (LQ) admission policy, which serves as a theoretical benchmark for the best-case collision-free channels.

To the best of our knowledge, this is the first population-based framework for multi-sensor remote estimation that accounts for sensor availability dynamics, energy constraints, and collision-prone channels, offering new perspectives for designing energy-efficient and reliable estimation systems.

II. Notation

We use the following notation. The set of real numbers is denoted by $\mathbb{R}$. The set of non-negative integers is denoted by $\mathbb{Z}_{\geq 0}$. The notation $\lfloor x \rceil$ represents the rounding of $x \in \mathbb{R}$ to the nearest integer. Sets are denoted by calligraphic uppercase letters, and their cardinality by $|\cdot|$. The notation $x \in \mathbb{R}^d$ represents a column vector (unless stated otherwise) of dimension d. The transpose of matrix A is denoted by $A^\top$, while its trace is denoted by $\mathrm{tr}(A)$. The d-dimensional identity (zero) matrix is denoted by I_d ($\mathbf{0}_d$). The notation $A \succeq 0$ states that matrix A is positive semidefinite. With $\Pr(\cdot)$ we refer to the probability of an event. By $\mathbb{E}[\cdot]$ we denote the expected value of a random variable.

III. System Model

The overall framework of the system for multisensor remote state estimation over an unreliable wireless channel is depicted in Fig. 1. In what follows, each component is discussed separately.

A. Dynamical system

Consider a discrete-time linear time-varying (LTV) dynamical system

$$x_{k+1} = A_k x_k + w_k, \tag{1}$$

where k is the discrete time index, $x_k \in \mathbb{R}^d$ is the system state, and $w_k \sim \mathcal{N}(0, \Sigma_k^w)$ is a zero mean Gaussian process noise with covariance $\Sigma_k^w \succeq 0$. The real-valued state-transition matrices $A_k \in \mathbb{R}^{d\times d}$ are assumed to be uniformly bounded.

B. Sensor participation and observation model

The dynamical system can be observed independently by multiple homogeneous[1] sensors. Let $\mathcal{V} = \{1, \dots, \bar{n}\}$ denote the (finite) set of all available sensors. Define the *activity status* for each sensor $i \in \mathcal{V}$ as:

$$\sigma_{i,k} := \begin{cases} 1, & \text{sensor } i \text{ is active at time } k, \\ 0, & \text{sensor } i \text{ is idle at time } k, \\ -1, & \text{sensor } i \text{ is depleted at time } k. \end{cases} \tag{2}$$

The *active sensor set* at time k is $\mathcal{A}_k := \{i \in \mathcal{V} : \sigma_{i,k} = 1\}$, containing sensors that acquire and transmit measurements

$$y_{i,k} = C_{i,k} x_k + v_{i,k}, \tag{3}$$

where $C_{i,k} \in \mathbb{R}^{m_i \times d}$ and $v_{i,k} \sim \mathcal{N}(0, \Sigma_{i,k}^v)$, with all noises mutually independent. The aggregated matrix $C_k^{\mathcal{A}} := \left[C_{i,k}\right]_{i\in\mathcal{A}_k}$ should be such that $(A_k, C_k^{\mathcal{A}})$ is observable for all k. The *idle sensor set* is $\mathcal{I}_k := \{i \in \mathcal{V} : \sigma_{i,k} = 0\}$, consisting of sensors with sufficient power that remain silent but can be activated. The *depleted sensor set* is $\mathcal{D}_k := \{i \in \mathcal{V} : \sigma_{i,k} = -1\}$, containing sensors unable to transmit due to insufficient energy. The number of sensors in each category is $n_k^{\mathcal{A}} := |\mathcal{A}_k|$, $n_k^{\mathcal{I}} := |\mathcal{I}_k|$, and $n_k^{\mathcal{D}} := |\mathcal{D}_k|$. Each sensor is in one of these states, with transitions governed by energy dynamics and the controller's activation decisions. The state of sensor's $i \in \mathcal{V}$ energy is denoted by $E_{i,k} \in [0, E_{\max}]$. Active sensors $i \in \mathcal{A}_k$ discharge according to

$$E_{i,k+1} = E_{i,k} e^{-\lambda_d \Delta_t}, \tag{4}$$

where λ_d is the discharge rate of the sensor and Δ_t is the time duration between the time indices k and $k+1$. The sensor becomes depleted when $E_{i,k+1} \leq E_{\min}$. Depleted sensors $i \in \mathcal{D}_k$ recharge as

$$E_{i,k+1} = E_{i,k} + (E_{\max} - E_{i,k})\big(1 - e^{-\lambda_r \Delta_t}\big), \tag{5}$$

where λ_r is the recharge rate of the sensors, which become idle once $E_{i,k+1} \geq E_{\text{thr}}$. Idle sensors continue charging until activated by the controller. We require $0 \leq E_{\min} \leq E_{\text{thr}} < E_{\max}$ to ensure feasible transitions. The exponential forms in (4)–(5) are standard first-order RC battery models [16].

In what follows, we provide a sensor admission model that evolves over time according to

$$n_{k+1} = W_k n_k + B_k u_k, \tag{6}$$

where $n_k := [n_k^{\mathcal{A}},\ n_k^{\mathcal{I}},\ n_k^{\mathcal{D}}]^\top$ denotes the number of active, idle, and depleted sensors, and $u_k := a_k n_k^{\mathcal{I}}$ admits a fraction a_k of idle sensors. The transition matrix W_k and the corresponding control matrix B_k are given by

$$W_k := \begin{bmatrix} 1-\delta_k & 0 & 0 \\ 0 & 1 & \rho_k \\ \delta_k & 0 & 1-\rho_k \end{bmatrix}, \quad B_k := \begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix}, \tag{7}$$

where $\delta_k := \frac{1}{n_k^{\mathcal{A}}}\big|\{i \in \mathcal{A}_k : E_{i,k+1} \leq E_{\min}\}\big|$ and $\rho_k := \frac{1}{n_k^{\mathcal{D}}}\big|\{i \in \mathcal{D}_k : E_{i,k+1} \geq E_{\text{thr}}\}\big|$ denote the fractions of sensors transitioning from active to depleted and from depleted to idle, respectively. The total number of sensors is conserved, *i.e.,* $n_k^{\mathcal{A}} + n_k^{\mathcal{I}} + n_k^{\mathcal{D}} = \bar{n}$.

[1] In our framework, *homogeneous sensors* refer to sensors that observe the same system state and share identical energy dynamics, *i.e.,* they follow the same discharging/recharging dynamics. This implies that all sensors are interchangeable from the perspective of the admission controller.

C. Unreliable wireless channel model

We model the wireless channel using a signal-to-interference-plus-noise (SINR) threshold model as in [14], [17], [18]. At time k, each active sensor $i \in \mathcal{A}_k$ transmits its measurement $y_{i,k}$ to the remote estimator. The reception is successful if

$$\gamma_{i,k} := \begin{cases} 1, & \text{if } \mathrm{SINR}_{i,k} \geq \beta, \\ 0, & \text{otherwise,} \end{cases} \tag{8}$$

where $\beta > 0$ is the decoding threshold depending on the modulation and coding scheme, and

$$\mathrm{SINR}_{i,k} = \frac{\pi h_{i,k}}{\sigma^2 + \sum_{j \in \mathcal{A}_k \setminus \{i\}} \pi h_{j,k}}, \tag{9}$$

where π is the transmit power (assumed common for all sensors due to homogeneity), $h_{i,k}$ the channel power gain, and σ^2 the noise power. We assume a Rayleigh fading channel with $h_{i,k} \sim \mathrm{Exp}(\lambda)$ i.i.d. for all i, where $1/\lambda$ is the average channel gain. The success probability admits the closed-form expression

$$\Pr(\gamma_{i,k} = 1 \mid n_k^{\mathcal{A}}) = p_0 \, (1+\beta)^{-(n_k^{\mathcal{A}}-1)} =: p(n_k^{\mathcal{A}}), \tag{10}$$

where $p_0 := \exp\big(-\beta\lambda\sigma^2/\pi\big)$. Notice that, the success probability decreases exponentially with the number of simultaneously active sensors. Let $\mathcal{R}_k := \{i \in \mathcal{A}_k : \gamma_{i,k} = 1\}$ denote the set of successfully received measurements, with cardinality $n_k^{\mathcal{R}} := |\mathcal{R}_k|$. Conditional on the number of active sensors $n_k^{\mathcal{A}}$, the success indicators $\gamma_{i,k}$ are conditionally independent Bernoulli random variables with success probability $p(n_k^{\mathcal{A}})$ as given in (10). Consequently, the number of successfully received measurements follows a binomial distribution, *i.e.,* $n_k^{\mathcal{R}} \sim \mathrm{Binomial}\big(n_k^{\mathcal{A}}, p(n_k^{\mathcal{A}})\big)$.

D. Remote estimator

The remote estimator maintains an estimate $\hat{x}_{k|k}$ and error covariance $P_{k|k}$ at each time step k. This estimate is obtained using the set of successfully received measurements $\{y_{\ell,k} : \ell \in \mathcal{R}_k\}$ at each time k. The successfully received measurements that arrived at time k are given by:

$$y_k^{\mathcal{R}} = C_k^{\mathcal{R}} x_k + v_k^{\mathcal{R}}, \tag{11}$$

where $y_k^{\mathcal{R}} \in \mathbb{R}^m$ stacks vertically the observations from the sensors whose measurements have been successfully received (resp. for $C_k^{\mathcal{R}}$, $v_k^{\mathcal{R}}$), with $m := \sum_{\ell \in \mathcal{R}_k} m_i$. Then, the remote estimator runs Kalman Filter updates with intermittent measurements conditioned on $\mathcal{R}_k$, as:

$$\hat{x}_{k|k-1} = A_{k-1}\, \hat{x}_{k-1|k-1}, \tag{12a}$$
$$P_{k|k-1} = A_{k-1} P_{k-1|k-1} A_{k-1}^{\top} + \Sigma_k^w, \tag{12b}$$
$$M_k = C_k^{\mathcal{R}} P_{k|k-1} (C_k^{\mathcal{R}})^{\top} + \Sigma_k^v, \tag{12c}$$
$$K_k = P_{k|k-1} (C_k^{\mathcal{R}})^{\top} M_k^{-1}, \tag{12d}$$
$$\hat{x}_{k|k} = \hat{x}_{k|k-1} + K_k \big(y_k^{\mathcal{R}} - C_k^{\mathcal{R}} \hat{x}_{k|k-1}\big), \tag{12e}$$
$$P_{k|k} = \big(I - K_k C_k^{\mathcal{R}}\big) P_{k|k-1}, \tag{12f}$$

while for $\mathcal{R}_k = \varnothing$ then $\hat{x}_{k|k} = \hat{x}_{k|k-1}$ and $P_{k|k} = P_{k|k-1}$. Note that, the number of measurements successfully received by the remote estimator is $n_k^{\mathcal{R}} \leq n_k^{\mathcal{A}}$ for all k. Upon computing $P_{k|k}$, the remote estimator computes a running error reference and provide a lightweight feedback about the estimation error by

$$P_k^{\mathrm{ref}} = \frac{1-\alpha}{\alpha} P_{k-1}^{\mathrm{ref}} + \mathrm{tr}(P_{k|k}), \tag{13}$$

with $P_{-1}^{\mathrm{ref}} = 0$ and $\alpha \in (0,1)$. The relative deviation is

$$\varepsilon_k = \frac{\mathrm{tr}(P_{k|k})}{P_k^{\mathrm{ref}}} \in [0,1], \tag{14}$$

which is uniformly quantized into 2^b levels to produce the feedback $z_k = \mathtt{Q}(\varepsilon_k)$, where $\mathtt{Q}(\cdot)$ is a b-bit quantizer over $[0,1]$.

E. Admission controller

An admission controller, broadcasts the control signal $u_k = a_k n_k^{\mathcal{I}}$, to enforce idle sensors to become active at time k. The number of sensors that can be admitted at time k is constrained by the current idle sensor population. Accordingly, the feasible set of admission decisions is

$$\mathcal{U}(n_k) := \{u_k \in \mathbb{Z}_{\geq 0} : 0 \leq u_k \leq n_k^{\mathcal{I}}\}. \tag{15}$$

Let $\mathcal{S} := \{n \in \mathbb{Z}_{\geq 0}^3 : n^{\mathcal{A}} + n^{\mathcal{I}} + n^{\mathcal{D}} = \bar{n}\}$ denote the feasible state space. The set of admission policies is denoted by $\mathcal{G}$ and consists of deterministic Markov feedback policies $g = \{g_k\}_{k=0}^{N-1}$, where each decision rule

$$g_k : \mathcal{S} \longmapsto \mathbb{Z}_{\geq 0}, \quad g_k(n) \in \mathcal{U}(n), \quad \forall n \in \mathcal{S},$$

maps the current state $n_k \in \mathcal{S}$ to a feasible admission decision. Given a policy $g \in \mathcal{G}$, the closed-loop dynamics evolve according to

$$n_{k+1}^g = W_k n_k^g + B_k g_k(n_k^g), \tag{16}$$

for $k = 0, 1, \ldots, N-1$, with initial condition $n_0 \in \mathcal{S}$. Hereinafter, we omit the superscript g when there is no ambiguity.

F. Optimization Problem

In this section, we formulate the optimization problem to capture the fundamental tradeoff in remote estimation over an unreliable wireless channel, that is, the admission controller must determine how many idle sensors to activate so as to balance estimation accuracy, network collision, and energy consumption. To this end, we define the stage cost as follows

$$\ell_k(n_k, u_k) = q_1 \mathbb{E}\Big[(n_k^{\mathcal{R}} - n_k^{\mathcal{R}^*})^2 | n_k^{\mathcal{A}}\Big] + q_2 u_k^2, \tag{17}$$

where $q_1, q_2 > 0$. At each step, the cost penalizes the deviation between the expected number of successfully received packets and the desired target $n_k^{\mathcal{R}^*}$, as well as the admission effort required for sensor activation. Since $n_k^{\mathcal{R}}$ is a Binomial random variable, its conditional mean is given by

$$\begin{aligned} \mathbb{E}\Big[(n_k^{\mathcal{R}} - n_k^{\mathcal{R}^*})^2 | n_k^{\mathcal{A}}\Big] &= \mathrm{Var}(n_k^{\mathcal{R}} | n_k^{\mathcal{A}}) + \big(\mathbb{E}[n_k^{\mathcal{R}} | n_k^{\mathcal{A}}] - n_k^{\mathcal{R}^*}\big)^2 \\ &= n_k^{\mathcal{A}} p(n_k^{\mathcal{A}})\big(1 - p(n_k^{\mathcal{A}})\big) + \big(n_k^{\mathcal{A}} p(n_k^{\mathcal{A}}) - n_k^{\mathcal{R}^*}\big)^2, \end{aligned} \tag{18}$$

where the reference target $n_k^{\mathcal{R}^*} = n_k^{\mathcal{A}^*} p(n_k^{\mathcal{A}^*})$ is defined as the conditional mean number of successful packets at the desired level $n_k^{\mathcal{A}^*}$. The reference target (desired active population) $n_k^{\mathcal{A}^*}$ is calculated based on the estimation error and its evolution, according to z_k. In particular, the admission control decodes the received signal z_k as $\hat{\varepsilon}_k = \mathtt{Q}^{-1}(z_k)$, where $\mathtt{Q}^{-1}(\cdot)$ is the inverse quantization operator. Here, for simplicity of exposition, we assume that the admission controller computes a mapping from $\hat{\varepsilon}_k$ into $n_k^{\mathcal{A}^*}$ as:

$$n_k^{\mathcal{A}^*} := \left\lfloor 1 + \hat{\varepsilon}_k (n_k^{\mathcal{I}} - 1) \right\rceil. \tag{19}$$

It is worth noting that this policy could be replaced by a more adaptive mapping from $\hat{\varepsilon}_k$ to $n_k^{\mathcal{A}^*}$, for example using proportional updates or other more sophisticated control strategies. However, designing such adaptive policies is beyond the scope of this work and is left for future research.

The stage cost in (17) can be equivalently written in quadratic form as

$$\ell_k(n_k, u_k) = n_k^\top Q_k(n_k^{\mathcal{A}}) n_k + 2 q_k^\top (n_k^{\mathcal{A}}) n_k + u_k^\top R_k u_k + \gamma_k,$$

where $Q_k(n_k^{\mathcal{A}}) = \operatorname{diag}\big(q_1 (p_k^{\mathcal{A}})^2, 0, 0\big)$ and $q_k(n_k^{\mathcal{A}}) = \left[\, \frac{q_1}{2}\big(p_k^{\mathcal{A}}(1-p_k^{\mathcal{A}}) - 2 n_k^{\mathcal{R}^*} p_k^{\mathcal{A}}\big),\ 0,\ 0 \,\right]^\top$. Here, for notational simplicity, we write $p_k^{\mathcal{A}}$ in place of $p(n_k^{\mathcal{A}})$, $R_k = q_2$, and $\gamma_k = q_1 (n_k^{\mathcal{R}^*})^2$. Then, the N–stage cost is defined as

$$J_N := \sum_{k=0}^{N-1} \ell_k(n_k, u_k) + n_N^\top Q_f n_N, \tag{20}$$

where $Q_f \succeq 0$ is a terminal weight penalizing the final state.

***Problem** 1:* The finite-horizon optimal admission problem is to choose an admissible sequence of control actions $\{u_k\}_{k=0}^{N-1}$ with $u_k \in \mathcal{U}_k(n_k)$ that solve

$$\begin{aligned} \min_{\{u_k\}_{k=0}^{N-1}} \quad & J_N := \sum_{k=0}^{N-1} \ell_k(n_k, u_k) + n_N^\top Q_f n_N \\ \text{subject to} \quad & n_{k+1} = W_k n_k + B_k u_k, \quad n_0 \text{ given.} \end{aligned}$$

In the admission control problem above, decisions are constrained by the number of idle sensors, network dynamics, and the wireless success probability, which decreases with simultaneous transmissions. The main difficulty stems from (10), as $p(n_k^{\mathcal{A}})$ induces a nonlinear dependence of the stage cost on the number of active sensors. To make analytical progress, we study two complementary cases:

- Case.1: A constant success probability $p(n_k^{\mathcal{A}}) \equiv p_0$ which provides analytical insight and serves as a best-case benchmark.
- Case.2: The general success probability $p(n_k^{\mathcal{A}})$ as defined in (10), which allow us to compute the optimal policy numerically via $n_k^{\mathcal{A}}$.

IV. Problem Solution

Let $V_k(n_k)$ be the minimal cost-to-go from time k onward starting at state n_k, given that an optimal policy g_t^*, $t = 0, 1, \ldots, k-1$, has been applied to reach n_k, defined by

$$V_k(n_k) = \min_{\{u_t\}_{t=k}^{N-1}} \sum_{t=k}^{N-1} \ell_k(n_t, u_t) + n_N^\top Q_f n_N. \tag{21}$$

By the Bellman's principle of optimality [19], the deterministic dynamic programming recursion is

$$V_N(n_N) = n_N^\top Q_f n_N, \tag{22a}$$

$$\begin{aligned} V_k(n_k) = \min_{u_k \in \mathcal{U}(n_k)} & \big\{ \ell(n_k, u_k) \\ & + V_{k+1}(W_k n_k + B_k u_k) \big\}, \end{aligned} \tag{22b}$$

for $k = N-1, N-2, \ldots, 0$. The dynamic programming recursion (22) relates the value functions $V_k(\cdot)$ and $V_{k+1}(\cdot)$, and is solved backward in time yielding $V_N(\cdot), \ldots, V_1(\cdot)$.

A. LQR solution

In this section, we consider the case where the channel success probability is constant, $p(n_k^{\mathcal{A}}) \equiv p_0$ (*i.e.,* Case.1). The problem reduces to a finite-horizon linear quadratic regulator (LQR), for which a closed-form solution is available. The following theorem is derived by allowing the admission decision u_k to take real values in the interval $[0, n_k^{\mathcal{I}}]$.

Theorem 1: Assume the success probability is constant, $p(n_k^{\mathcal{A}}) \equiv p_0$, that at each time k a desired active level $n_k^{\mathcal{A}^*}$ is known, and that the admission decision u_k can take real values in the interval $[0, n_k^{\mathcal{I}}]$.

(a) (Unconstrained) The solution of the dynamic programming equation (22) has a quadratic form

$$V_k(n) = n^\top S_k n + 2 s_k^\top n + c_k, \quad k = 0, 1, \ldots, N, \tag{23}$$

with terminal condition $S_N = Q_f \succeq 0$, $s_N = 0$, and $c_N = 0$. The matrices satisfy $S_k \succeq 0$ for all k, while $s_k \in \mathbb{R}^{|n|}$ and $c_k \in \mathbb{R}$. The unconstrained optimal admission decision is

$$u_k^{\mathrm{LQ}}(n) = -L_k n - h_k, \tag{24}$$

where the gains are $L_k = H_k^{-1} B_k^\top S_{k+1} W_k$, and $h_k = H_k^{-1} B_k^T s_{k+1}$ with $H_k = R_k + B_k^\top S_{k+1} B_k$. The Riccati equations satisfy the backward recursions

$$\begin{aligned} S_k &= Q_k + (W_k - B_k L_k)^\top S_{k+1} (W_k - B_k L_k) + L_k^\top R_k L_k, \\ s_k &= q_k + (W_k - B_k L_k)^\top (s_{k+1} - S_{k+1} B_k h_k) + L_k^\top R_k h_k, \\ c_k &= c_{k+1} + \gamma_k + h_k^\top R_k h_k - (B_k h_k)^\top S_{k+1} (B_k h_k) \\ &\quad - 2 h_k^\top B_k^\top s_{k+1}. \end{aligned}$$

(b) (Constrained) Since the admission decision is restricted to the interval $0 \le u_k \le n_k^{\mathcal{I}}$, the optimal admission decision is obtained by projecting the unconstrained policy of part (a) onto the feasible set

$$u_k^*(n) = \Pi_{\mathcal{U}}\big(u_k^{\mathrm{LQ}}(n)\big), \tag{25}$$

where for any scalar $z \in \mathbb{R}$,

$$\Pi_{\mathcal{U}}(z) := \min\big\{ \max\{z, 0\}, n_k^{\mathcal{I}} \big\}. \tag{26}$$

Proof: The proof of part (a) follows the classical one and hence it is omitted. For part (b), notice that the minimization in u_k is strictly convex, so the unconstrained admission policy $u_k^{\mathrm{LQ}}(n)$ in (24) is the unique minimizer. Imposing the feasibility constraint $0 \leq u_k \leq n_k^{\mathcal{I}}$ reduces to minimizing a strictly convex quadratic function over a compact interval. By standard convex optimization results (see [20]), the unique solution is the projection of $u_k^{\mathrm{LQ}}(n)$ onto the interval $[0, n_k^{\mathcal{I}}]$, which gives (26). ∎

Remark 1: Theorem 1 gives the optimal admission policy in closed form assuming $u_k \in [0, n_k^{\mathcal{I}}]$. In practice, since sensors are integer, $u_k^*(n)$ is rounded to the nearest value in $\mathcal{U}(n_k) = 0, 1, \ldots, n_k^{\mathcal{I}}$. This real-valued solution serves as a benchmark for evaluating the numerical dynamic programming policy in the collision-aware model.

B. Numerical solution

In this section, we consider the case where the channel success probability $p(n_k^{\mathcal{A}})$ is given by (10) (*i.e.,* Case.2). Since the total sensor population is fixed at $\bar{n}$, the depleted population is determined by the relation

$$n_k^{\mathcal{D}} = \bar{n} - n_k^{\mathcal{A}} - n_k^{\mathcal{I}}. \tag{27}$$

Thus, we can represent the system state in reduced form as $\tilde{n}_k := \begin{bmatrix} n_k^{\mathcal{A}} & n_k^{\mathcal{I}} \end{bmatrix}^\top$. Substituting (27) into (6) yields the reduced state dynamics in affine form

$$\tilde{n}_{k+1} = \tilde{W}_k n_k + \tilde{B}_k u_k + d_k, \tag{28}$$

where

$$\tilde{W}_k := \begin{bmatrix} 1-\delta_k & 0 \\ -\rho_k & 1-\rho_k \end{bmatrix}, \quad \tilde{B}_k := \begin{bmatrix} 1 \\ -1 \end{bmatrix}, \quad d_k := \begin{bmatrix} 0 \\ \rho_k \bar{n} \end{bmatrix},$$

with admissible actions $u_k \in \mathcal{U}(\tilde{n}_k) := \{0, 1, \ldots, n_k^{\mathcal{I}}\}$. The reduced stage cost is defined through the mapping $\Phi : \mathbb{R}^2 \longmapsto \mathbb{R}^3$,

$$\Phi(\tilde{n}_k) := \begin{bmatrix} n_k^{\mathcal{A}}, & n_k^{\mathcal{I}}, & \bar{n} - n_k^{\mathcal{A}} - n_k^{\mathcal{I}} \end{bmatrix}^\top, \tag{29}$$

which reconstructs the full state from $\tilde{n}_k$. Then, the stage cost in reduced form is

$$\tilde{\ell}_k(\tilde{n}_k, u_k) = \ell_k(\Phi(\tilde{n}_k), u_k). \tag{30}$$

The reduced dynamic programming recursion is given by

$$\tilde{V}_N(\tilde{n}_N) = \Phi(\tilde{n}_N)^\top Q_f \Phi(\tilde{n}_N), \tag{31a}$$

$$\begin{aligned} \tilde{V}_k(\tilde{n}_k) = \min_{u_k \in \mathcal{U}(\tilde{n}_k)} \big\{ & \tilde{\ell}_k(\tilde{n}_k, u_k) \\ & + \tilde{V}_{k+1}(\tilde{W}_k \tilde{n}_k + \tilde{B}_k u_k + d_k) \big\}, \end{aligned} \tag{31b}$$

for $k = N-1, N-2, \ldots, 0$. Note that, (31a) defines the reduced terminal cost in terms of the reduced state $\tilde{n}$, by evaluating the terminal cost (22a) at the corresponding state $n = \Phi(\tilde{n})$. For numerical implementation, it is convenient to expand this expression in quadratic form. Writing the mapping Φ in (29) as

$$\Phi(\tilde{n}) := T\tilde{n} + t, \quad T = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ -1 & -1 \end{bmatrix}, \quad t = \begin{bmatrix} 0 \\ 1 \\ \bar{n} \end{bmatrix},$$

we obtain

$$\begin{aligned} \tilde{V}_N(\tilde{n}) &= (T\tilde{n} + t)^\top Q_f (T\tilde{n} + t) \\ &= \tilde{n}^\top \tilde{Q}_f \tilde{n} + 2\tilde{q}_f^\top \tilde{n} + \gamma_f, \end{aligned} \tag{32}$$

where $\tilde{Q}_f = T^\top Q_f T$, $\tilde{q}_f = T^\top Q_f t$, and $\gamma_f = t^\top Q_f t$.

Proposition 1: Let $\{V_k\}_{k=0}^N$ denote the value functions of the dynamic programming recursion (22), and let $\{\tilde{V}_k\}_{k=0}^N$ denote the value functions of the reduced dynamic programming recursion (31). Then, for all $k \in \{0, 1, \ldots, N\}$, and all feasible reduced states $\tilde{n}$,

$$\tilde{V}_k(\tilde{n}) = V_k(\Phi(\tilde{n})). \tag{33}$$

The optimal admission policies of (22) and (31) coincide.

Proof: The mapping Φ defined in (29) is a bijection between the set of feasible reduced states $\tilde{n} = \begin{bmatrix} n_k^{\mathcal{A}} & n_k^{\mathcal{I}} \end{bmatrix}^\top$ satisfying $n^{\mathcal{A}} \geq 0$, $n^{\mathcal{I}} \geq 0$, and $n^{\mathcal{A}} + n^{\mathcal{I}} \leq \bar{n}$, and the set of feasible states $n = \begin{bmatrix} n_k^{\mathcal{A}} & n_k^{\mathcal{I}} & n_k^{\mathcal{D}} \end{bmatrix}^\top$ with $n^{\mathcal{A}} + n^{\mathcal{I}} + n^{\mathcal{D}} = \bar{n}$. Since the admissible action set depends only on $n^{\mathcal{I}}$, we have that $\mathcal{U}(\tilde{n}) = \mathcal{U}(\Phi(\tilde{n}))$. The terminal costs (22a) and (31a) agree because

$$\tilde{V}_N(\tilde{n}) = \Phi(\tilde{n})^\top Q_f \Phi(\tilde{n}) = V_N(\Phi(\tilde{n})).$$

Now, suppose $\tilde{V}_{k+1}(\tilde{n}) = V_{k+1}(\Phi(\tilde{n}))$. Using the reduced stage cost (30) and dynamics (28), we obtain

$$\begin{aligned} \tilde{V}_k(\tilde{n}) &= \min_{u \in \mathcal{U}(\tilde{n})} \{\tilde{\ell}(\tilde{n}, u) + \tilde{V}_{k+1}(\tilde{W}_k \tilde{n} + \tilde{B}_k u + d_k)\} \\ &= \min_{u \in \mathcal{U}(\Phi(\tilde{n}))} \{\ell(\Phi(\tilde{n}), u) + V_{k+1}(W_k \Phi(\tilde{n}) + B_k u)\} \\ &= V_k(\Phi(\tilde{n})). \end{aligned}$$

Hence, by backward induction, $\tilde{V}_k(\tilde{n}) = V_k(\Phi(\tilde{n}))$ holds for all k. Since the minimizing actions are preserved under Φ, the optimal admission policies of (22) and (31) coincide. ∎

Having established the aforementioned dynamic programming formulations, we now turn to numerical simulations to assess the performance of the proposed approach and illustrate the impact of the admission controller on the state estimation problem.

V. Numerical Results

To illustrate the performance of the proposed methods in Section IV, we run simulations with the discrete-time system (1) and $A_k = \begin{bmatrix} 1 & 1; & 0 & 1 \end{bmatrix}$ for all $k \geq 0$, under two modes: Mode A ($0 \leq k \leq 24$) and Mode B ($25 \leq k \leq 50$), with process noise covariances $\Sigma_A^w = \begin{bmatrix} 0.001 & 1; & 0 & 0.001 \end{bmatrix}$ and $\Sigma_B^w = \begin{bmatrix} 0.5 & 1; & 0 & 0.5 \end{bmatrix}$. The system is observed by $\bar{n} = 60$ homogeneous sensors with $C_{i,k} = \begin{bmatrix} 0.6 & 0 \end{bmatrix}$ and $\Sigma_{i,k}^v = 0.1$. Each sensor follows the discharge/recharge model in (4)–(5) with $\lambda_d = \lambda_r = 0.1$, $\Delta_t = 1$, and $E_{i,0}$ uniformly in $(0, 1)$, yielding the population dynamics in (6)–(7). Active sensors transmit over an unreliable channel with success probability (10), using $\beta = 0.2$, $\lambda = 0.8$, $\pi = 10$, and $\sigma = 0.01$, so that $n_k^{\mathcal{R}} = n_k^{\mathcal{A}^*} p(n_k^{\mathcal{A}^*})$. The remote estimator feeds back a quantized error measure $z_k = \mathsf{Q}(\varepsilon_k)$ (using 4 bits), from which the reference $n_k^{\mathcal{A}^*}$ is computed via (19). The stage cost weights are $q_1 = 1000$ and $q_2 = 100$.

Fig. 2 shows the evolution of sensor populations, control $u_k^\star$, and estimation performance $\mathrm{tr}(P_{k|k})$. When the estimation error is low, fewer sensors are activated; as it increases (notably in Mode B), more sensors are admitted to reduce uncertainty (e.g., 11 sensors at $k = 27$). The state estimate tracks the true state despite packet losses. Fig. 3 reports the average cost $\bar{J} = \frac{1}{\mu}\sum_{i=1}^{\mu} J_N^{(i)}$ over $\mu = 50$ runs for different p_0. The closed-form solution (Section IV-A), which neglects collisions, provides a lower bound. For the numerical solution (Section IV-B), with $p(n_k^{\mathcal{A}}) = p_0(1+\beta)^{-(n_k^{\mathcal{A}}-1)}$, stronger collision effects lead to higher cost.

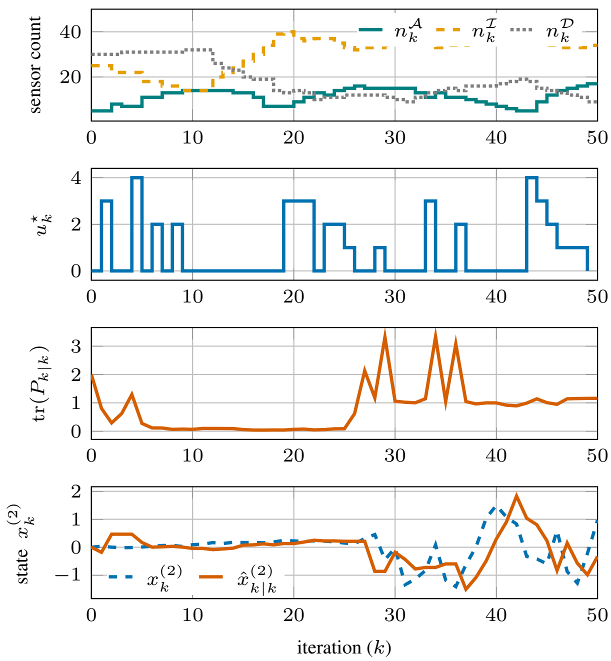

Fig. 2. System evolution. From top to bottom: number of active ($n_k^{\mathcal{A}}$), idle ($n_k^{\mathcal{I}}$), and depleted ($n_k^{\mathcal{D}}$) sensors; optimal admission control signal $u_k^{\star}$; estimation error covariance; and estimated state.

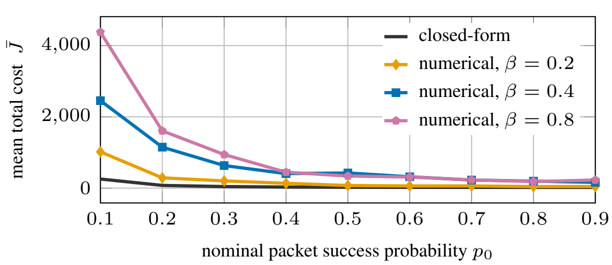

Fig. 3. Comparison of the closed-form and numerical DP admission controller.

VI. Conclusions

This paper addressed remote state estimation over a shared unreliable wireless channel with energy-constrained sensors. Unlike traditional approaches assuming fixed participation, we modeled sensor admission control as a dynamic decision process balancing estimation accuracy, collisions, and energy depletion. A population-based model captures the evolution of active, idle, and depleted sensors alongside a probabilistic channel reflecting collision-induced packet losses. The finite-horizon admission control problem is solved numerically, and a closed-form linear–quadratic policy provides structural insight into the trade-off between communication effort and estimation performance under fixed transmission reliability.

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